

Navigating with the Kamal - Northern Hemisphere

How did Arabian sailors navigate at sea?

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KEYWORDS

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Meridian, Longitude, Earth, Geography, Latitude, Pole height, Astronomy, Arabia, Celestial navigation, Polaris, Kamal, History, Stars, Equator, Countries, North Star, Navigation

B	
	CATEGORY
	Astrometry and celestial mechanics
0	LOCATION Small Indoor Setting (e.g. classroom)
•••	AGE 14 - 16
	LEVEL Middle School
	TIME 1h30
\bigcirc	GROUP Group





With this activity, the students will learn that - Celestial navigation and the corresponding tools have already been developed many centuries ago. - The kamal is a simple tool to measure the elevations of stars. - With the kamal, we can easily determine our latitude on earth.



LEARNING OBJECTIVES

- Students will build their own historical navigational instrument to understand how medieval mariners used the stars for navigation.
- They will use it to determine their latitude on earth and so understand how simple and accurate this device is.
- In the course of the activity, the students will learn to find Polaris, the North Star, to be able to determine the cardinal directions during the night, which provides them with basic knowledge for navigating the seas.



Latitude and longitude

Figure 1: Illustration of how the latitudes and longitudes of the Earth are defined (Credits: Peter Mercator, djexplo, CCO).



Any location in an area is defined by two coordinates. The surface of a sphere is a curved area, and using directions like up and down is not useful, because the surface of a sphere has neither a beginning nor an ending. Instead, we can use spherical polar coordinates originating from the centre of the sphere, which has a fixed radius (Figure 1). Two angular coordinates remain, which for the Earth are called the latitude and the longitude. The axis of rotation is also the symmetry axis. The North Pole is defined as the point where the theoretical axis of rotation coincides with the surface of the sphere, and the Earth rotates in a counter-clockwise direction when the pole is viewed from above. The opposite point is the South Pole. The equator is defined as the great circle halfway between the poles.

The latitudes are circles parallel to the equator. They are counted from 0° at the equator to 390° at the poles. The longitudes are great circles connecting the two poles of the Earth. For a given position on Earth, the longitude going through the zenith, which is the point directly above, is called the meridian. This is the line that the Sun apparently crosses at local noon. The origin of this coordinate is defined as the meridian of Greenwich, where the Royal Observatory of England is located. From there, longitudes are counted from 0° to 3180°.

Example: Heidelberg in Germany is located at 49.4° North and 8.7° East.

Elevation of the pole (pole height)

If we project the terrestrial coordinate system of latitudes and longitudes in the sky, we get the celestial equatorial coordinate system. The Earth's equator becomes the celestial equator and the geographical poles are extrapolated to build the celestial poles. If we were to take a photograph of the northern sky with a long exposure, we would see from the trails of the stars that they all revolve about a common point, which is the northern celestial pole (Figure 2).

In the northern hemisphere, there is a moderately bright star near the celestial pole, which is the North Star or Polaris. It is the brightest star in the Little Bear constellation, or Ursa Minor (Figure 3). In the present era, Polaris is less than a degree off. However, 1000 years ago, it was 8° away from the pole. Therefore, today, we can use it as a proxy for the position of the celestial north pole. At the southern celestial pole, there is no such star that can be observed with the naked eye. Other procedures have to be applied to find it.

Figure 2: Trails of stars at the sky after an exposure time of approximately 2 hours (Credit: Ralph Arvesen, Live Oak star trails, https://www.flickr.com/photos/rarvesen/9494908143, https://creativecommons.org/licenses/by/2.0/legalcode)



Figure 3: Configuration of the two constellations Ursa Major (Great Bear) and Ursa Minor (Little Bear) in the northern sky. Polaris, the North Star, which is close to the true celestial north pole, is the brightest star in Ursa Minor (Credit: Bonč, https://commons.wikimedia.org/wiki/File:Ursa_Major_- *Ursa_Minor* -_Polaris.jpg, 'Ursa Major – Ursa Minor – Polaris', based on https://commons.wikimedia.org/wiki/File:Ursa_Major_constellations.jpg, colours inverted by Markus Nielbock, https://creativecommons.org/licenses/by-sa/3.0/legalcode).



If we stood exactly at the geographical North Pole, Polaris would always be directly overhead. We can say that its elevation would be (almost) 90°. This information introduces the horizontal coordinate system (Figure 4), which is a natural reference we use every day. We, the observers, are the origin of that coordinate system located on a flat plane, whose edge is the horizon. The sky is imagined as a hemisphere above. The angle between an object in the sky and the horizon is the altitude or elevation. The direction within the plane is given as an angle between 0° and 360°, the azimuth, which is usually measured clockwise from the north. In navigation, this is also called the bearing. The meridian is the line that connects north and south at the horizon and passes the zenith.

Figure 4: Illustration of the horizontal coordinate system. The observer is the origin of the coordinates assigned as the azimuth and altitude or elevation (Credit: TWCarlson, https://commons.wikimedia.org/wiki/File:Azimuth-Altitude_schematic.svg, 'Azimuth-Altitude schematic', https:// creativecommons.org/licenses/by-sa/3.0/legalcode).



For any other position on Earth, the celestial pole or Polaris would appear at an elevation less than 90°. At the equator, it would just appear at the horizon, i.e. at an elevation of 0°. The correlation between the latitude (North Pole = 90°, Equator = 0°) and the elevation of Polaris is no coincidence. Figure 5 combines all three mentioned coordinate systems. For a given observer at any latitude on Earth, the local horizontal coordinate system touches the terrestrial spherical polar coordinate system at a single tangent point. The sketch demonstrates that the elevation of the celestial north pole, also called the pole height, is exactly the northern latitude of the observer on Earth.

Figure 5: When the three coordinate systems (terrestrial spherical, celestial equatorial and local horizontal) are combined, it becomes clear that the latitude of the observer is exactly the elevation of the celestial pole, also known as the pole height (Credit: M. Nielbock, own work).



From this, we can conclude that if we measure the elevation of Polaris, we can determine our latitude on Earth with reasonable precision.

Triangles and trigonometry

The concept of the kamal relies on the relations within triangles. These are very simple geometric constructs that the ancient Greeks worked with. One basic rule is that the sum of all angles in a triangle is 180° or [®]. This depends on whether the angles are measured in degrees or radians. One radian is defined as the angle that is subtended by an arc whose length is the same as the radius of the underlying circle. A full circle measures 360° or 2[®].

The sides of a triangle and its angles are connected via trigonometric functions, e.g. sine, cosine and tangent. The easiest relations can be seen in right-angled triangles, where one of the angles is 90° or @/2.

Figure 6: A right-angled triangle with γ being the right angle (Credit: Dmitry Fomin, CCO).



The hypotenuse is the side of a triangle opposite the right angle. In Figure 6, it is c. The other sides are called legs or catheti. The leg opposite to a given angle is called the opposite leg, while the other is the adjacent leg. In a right-angled triangle, the relations between the legs and hypotenuse are expressed as trigonometric functions of the angles.

 $\sin \alpha = a/c = opposing \log / hypotenuse$ (Equation 1)

 $\cos \alpha = b/c = adjacent \log / hypotenuse (Equation 2)$

 $\tan \alpha = (\sin \alpha) / (\cos \alpha) = a/b = opposing leg / adjacent leg (Equation 3)$

The Pythagorean Theorem tells us something about the relations between the three legs of a right-angled triangle. It is named after the ancient Greek mathematician Pythagoras and states that the sum of the squares of the catheti is equal to the square of the hypotenuse.

 $c^{2} = a^{2} + b^{2}$ (Equation 4)

For general triangles, this expands to the law of cosines.

 $c^{2} = a^{2} + b^{2} - 2ab \cdot \cos y$ (Equation 5)

For γ =90°, it reduces to the Pythagorean Theorem.

Early navigation

Early seafaring peoples often navigated along coastlines before sophisticated navigational skills were developed and tools were invented. Sailing directions helped to identify coastal landmarks (Hertel, 1990). To some extent, their knowledge about winds and currents helped them to cross short distances, e.g. in the Mediterranean.

Soon, navigators realised that celestial objects, especially stars, can be used to maintain the course of a ship. Such skills have been mentioned in early literature like Homer's Odyssey, which is believed to date back to the 8th century BCE. There are accounts of ancient Phoenicians who were able to even leave the Mediterranean and ventured on voyages to the British coast and even several hundred miles south along the African coast (Johnson & Nurminen, 2009). A very notable and well-documented long-distance voyage has been mentioned by ancient authors and scholars like Strabo, Pliny and Diodorus of Sicily. It is the voyage of Pytheas, a Greek astronomer, geographer and explorer from Marseille who, around 300 BCE, apparently left the Mediterranean by passing Gibraltar and carried on north until the British Isles and beyond the Arctic Circle, where he possibly reached Iceland or the Faroe Islands, which he called Thule (Baker & Baker, 1997). Pytheas used a gnomon or sundial, which allowed him to determine his latitude and measure the time during his voyage (Nansen, 1911).

Sailing along a latitude

At these times, the technique of sailing along a parallel (of the equator) or latitude was based on observing circumpolar stars. The concept of latitudes in the sense of angular distances from the equator was probably not known. However, it was soon realised that when looking at the night sky, some stars within a certain radius around the celestial poles never set; these are circumpolar stars. When sailing north or south, sailors observe that the celestial pole changes, too, and with it, the circumpolar radius. Therefore, whenever navigators see the same star culminating, i.e. transiting the meridian, at the same elevation, they stay on the 'latitude'. For them, it was sufficient to realise the connection between the elevation of stars and their course. Navigators had navigational documents that listed seaports together with the elevation of known stars. In order to reach the port, they simply sailed north or south until they reached the corresponding latitude and then continued west or east.

Nowadays, the easiest way to determine one's latitude on Earth is to measure the elevation of the North Star, Polaris, as a proxy for the true celestial North Pole. In our era, Polaris is less than a degree off. However, 1000 years ago, it was 8° away from the pole.

The kamal

The kamal is a navigational tool invented by Arabian sailors in the 9th century CE (McGrail, 2001). Its purpose is to measure stellar elevations without the notion of angles. If you stretch out your arm, one finger subtends an angle. This method appears to have been the earliest technique to determine the elevation of stars. In the Arabian world, this 'height' is called isba (إصبع), which simply means finger. The corresponding angle is 1°36' (Malhão Pereira, 2003).

Figure 7: A simple wooden kamal. It consists of a surveying board and a cord with a line of knots (Credit: Bordwall https://commons.wikimedia.org/wiki/ File:Simple_Wooden_Kamal_(Navigation).jpg, 'Simple Wooden Kamal (Navigation)', https://creativecommons.org/licenses/by-sa/3.0/legalcode).



This method was standardised by using a wooden plate, originally sized roughly 5 cm \times 2.5 cm, with a cord attached to its centre. When held at various distances, the kamal subtends different angles between the horizon and the stars (Figure 8). Knots located at different positions along the cord denote the elevations of stars and, consequently, the latitude of various ports.

Figure 8: Illustration of how the kamal was used to measure the elevation of a star, in this case, Polaris. The lower edge was aligned to the horizon. Then, the distance between the eyes and the kamal was modified until the upper edge touched the star. The distance was set by knots tied into the cord that was held between the mouth and the kamal. The knots indicate the elevations of stars (Credit: M. Nielbock, https://commons.wikimedia.org/wiki/File:Kamal_Polaris_Dide.png, https:// creativecommons.org/licenses/by/4.0/legalcode).



When Vasco da Gama set out to find the sea passage from Europe to India in 1497, he stopped at the Eastern African port of Melinde (now, Malindi), where the local Muslim Sheikh provided him with a skilled navigator of the Indian Ocean to guide him to the shores of India. This navigator used a kamal for finding the sailing directions (Launer, 2009).

Since the latitudes the Arabian sailors crossed during their passages through the Arabian and Indian Seas are rather small, the mentioned size of the kamal is sufficient. For higher latitudes, the board must be bigger so that the cord is not too short to realise such angles.

Figure 9: Excerpt of a world map from 1502 showing the Indian Ocean. All sea routes from the Arabian Peninsula and India lie between the Tropic of Cancer and the Equator. The port of Melinde is indicated at the third flag from the top at the eastern African coast (Credit: Cantino Planisphere, 1502, Biblioteca Estense Universitaria, Modena, Italy, https://commons.wikimedia.org/wiki/ File:Cantino_planisphere_(1502).jpg, public domain).



The geometry of the kamal

To measure an angle φ with a kamal of height h, the distance between the eyes and the board held perpendicularly to the line of sight needed is l. This is realised by a knot in the cord on the side opposite to the kamal board. In this simple configuration, we get:

$I = h' / \tan \varphi' = h / (2 \cdot \tan(\varphi/2))$ (Equation 6)

Figure 10: Simplified geometry of the kamal, which subtends an angle φ between the horizon and Polaris. The kamal has a height labelled h. The length of the cord between the eyes and the kamal is labelled I (Credit: M. Nielbock, own work).



However, the length is measured with the cord between the teeth or just in front of the lips. The eyes and mouth are separated by the length d (Figure 11). The true length of the cord is then I, while I' is the distance between the eyes and the kamal board that defines the angle φ . This more realistic approach leads to the following equation:

(Equation 7)



We see that for d=0, we again get the simplified version above. The difference between I and I' can be a few centimetres. A realistic value is d=7 cm.

This geometry is accurate enough for uncertainties inherent to the measurement method. Note that it is always assumed that the kamal board is held at an angle perpendicular to the line of sight, not the cord. In addition, the horizon is assumed to be the mathematical one (Figure 5). This means that the dip of the visible horizon is neglected.

Figure 11: More realistic geometry of the kamal considering the difference in distance between the kamal on one side and the mouth and the eyes on the other. The distance between the mouth and the eyes is labelled d (Credit: M. Nielbock, own work).



Glossary

Apparent movement

Movement of celestial objects which, in fact, is caused by the rotation of the Earth.

Cardinal directions

Main directions, i.e. north, south, west and east

Circumpolar

Property of celestial objects that never set below the horizon.

Culmination

Passing the meridian of celestial objects. These objects attain their highest or lowest elevation there.

Diurnal

Concerning a period that is caused by the daily rotation of the Earth around its axis.

Elevation

Angular distance between a celestial object and the horizon.

Great circle

A circle on a sphere, whose radius is identical to the radius of the sphere.

Meridian

A line that connects north and south at the horizon via the zenith.

Pole height

Elevation of a celestial pole. Its value is identical to the latitude of the observer on earth.

Spherical polar coordinates

The natural coordinate system of a flat plane is Cartesian and measures distances in two perpendic¬ular directions (ahead, back, left, right). For a sphere, this is not very useful, because it has neither a beginning nor ending. Instead, the fixed point is the centre of the sphere. When projected outside from the central position, any point on the surface of the sphere can be determined by two angles, with one of them being related to the symmetry axis. This axis defines the two poles. In addition, there is the radius that represents the third dimension of space, which enables us to determine each point within a sphere. This defines the spherical polar coordinates. When defining points on the surface of a sphere, the radius stays constant.

Zenith Point in the sky directly above.

FULL DESCRIPTION

INTRODUCTION

It would be beneficial if the activity were to be discussed in the larger context of seafaring, e.g. in geography, history, literature, etc.

Tip: This activity could be combined with other forms of acquiring knowledge like oral presentations in history, literature or geography with navigation as the highlight. This would represent the field in a much more interactive way than what a teacher can achieve by summarising the facts.

Tip: There are excellent documentaries available on sea exploration and navigation that could be shown as an introduction.

Episode 2: Celestial Navigation (Duration: 4:39mins) https://youtu.be/ DoOuSo9qEll

How did early Sailors navigate the Oceans? | The Curious Engineer (Duration: 6:20mins) https://youtu.be/4DINhbkPiYY

Isn't that India? - Navigation at Sea I PIRATES (Duration: 5:56 mins) https:// youtu.be/OCPnmfe5PJ4

Navigation in the Age of Exploration (Duration: 7:05 mins) https://youtu.be/ X3Egmp94aZw

World Explorers in 10 Minutes (Duration: 9:59mins) https://youtu.be/iUkOfzhvMMs

Once upon a time ... man: The Explorers - The first navigators (Duration: 23:13 mins) https://youtu.be/KuryXLnHsEY

The Ancient Seamasters (Duration: 1:29:07mins) https://youtu.be/47kAtmYTCmY

Ask the students if they have any ideas about how long mankind has used ships to cross the oceans. One may point out the spread of Homo sapiens to islands and isolated continents like Australia.

Possible answers: We know for sure that ships have been used to cross large distances since 3,000 BCE or earlier. However, the early settlers in Australia must have found a way to cross the oceans around 50,000 BCE.

Ask the students what the benefits of trying to explore the seas could have been. Perhaps, someone knows historic cultures or peoples that were famous sailors. The teacher can support this with a few examples of ancient seafaring peoples, e.g. from the Mediterranean.

Possible answers: Finding new resources and food, trade, the spirit of exploration and curiosity.

Ask the students how they find their way to school every day. What supports their orientation so they don't get lost? As soon as reference points (buildings, traffic lights, bus stops, etc.) have been mentioned, ask the students how navigators were

able to find their way on the seas. In early times, people used sailing directions in connection to recognisable landmarks. But for this, the ships would have to stay close to the coast. Lighthouses improved the situation. Magnetic compasses have been a rather late invention, around the 11th century CE, and they were not used in Europe before the 13th century. So what could be used as reference points in the open sea? Probably the students will soon mention celestial objects like the Sun, the Moon and stars.

Tell the story of the kamal and Vasco da Gama, the discoverer of the direct passage from Europe to India. See the corresponding section in the background material and https://archive.org/stream/vascodagamahisvoOOtowl#page/136/ mode/2up http://www.heritage-history.com/? c=read&author=towle&book=dagama&story=king

ACTIVITY: BUILDING THE KAMAL

This can be done by the teacher prior to the activities or introduced as an additional exercise for the students. An instruction manual is available separately.

Material needed: - one piece of ply wood (preferred) or very stiff - card board (21 $cm \times 12 cm \times 4 mm$) - 50 cm of cord - Pencil - Ruler - Saw (for the wood) or scissors (for the cardboard), if the board has to be cut to fit the size needed - Drill (for the wood) or thick needle (for the cardboard)

The kamal was originally conceived as a navigational tool for low latitudes. Therefore, its size was relatively small, i.e. a few centimetres. This was enough to measure angles of 10° to 20° degrees above the horizon. For example, for a kamal of height 5 cm, a cord length of 20 cm yields an elevation measure of 15°. However, this relation is not linear. Therefore, for higher latitudes, a larger kamal board is needed. A good compromise is a height of 21 cm, while the width can be 12 cm. With these dimensions, the following relations hold. For very low latitudes, the kamal can be rotated by 90°, and the smaller width permits smaller cord lengths to reach the same angles.

Table 1: Dimensions and relations between the angles and lengths of a kamal according to Eq. (7). The distance between the eyes and the mouth is assumed to be d=7 cm.

Angle subtended (°) | Board height (cm)| Cord length (cm)| Board width (cm)| Cord length (cm) --- | --- | --- 30 | 21 | 41.6 | **12** | **25.1** 35 | 21 | 36.0 | **12** | **22.2** 40 | 21 | 31.9 | **12** | **20.0** 45 | **21** | **28.8** | 12 | 18.3 50 | **21** | **26.3** | 12 | 17.0 55 | **21** | **24.2** | 12 | 16.0 60 | **21** | **22.5** | 12 | 15.2 65 | **21** | **21.1** | 12 | 14.4 70 | **21** | **19.9** | 12 | 13.8

For each kamal, prepare a thin piece of ply wood (approx. 4 mm) of size 21 cm × 12 cm. If this is not available, a piece of very stiff cardboard of equal size can also be used. Determine the centre of the board by drawing or scratching two diagonal lines that connect opposite corners. Drill a hole through the centre that is big enough to permit the cord to fit through. It must also be small enough not to let it slide out again after a knot is tied.

Figure 12: The kamal after running the cord through the central hole (Credit: M. Nielbock, own work)



Tie a knot at one end of the cord and run it through the central hole of the board. The knot should block the cord from sliding through the hole.

Now add knots at distances from the board as indicated in Table 1. Be careful to keep the cord straight. You can restrict the number of knots according to the angular range needed for the activities. Remember that the elevation of Polaris corresponds to the latitude.

Fill out the table on the worksheet that lists the number of knots and the corresponding angles.

ACTIVITY: ANGELS IN THE SKY

Introduction

The worksheets contain Figure 2 (star trails). There are a few questions to be asked that can help students understand the concept of the apparent trajectories of stars.

Q: What does this picture show, in particular, where do the bright curved lines come from?

A: As the Earth rotates, the stars seem to revolve around a common point. This is the celestial pole. The long exposure enables visualisation of the path of the stars as trails.

Q: How does the picture show us that some stars do not set or rise during a full day?

A: Many trails can be followed to form a full circle. One rotation is 24 hours.

Q: Can you identify the star that is next to the celestial North Pole? In this picture, it should be close to the centre of rotation.

A: This is Polaris or the North Star. It is the star that produces the smallest trail close to the centre of the trails.

Q: Imagine you are at the terrestrial North Pole. Where would Polaris be in the sky? Where would it be if you stood at the equator? A: North Pole: zenith, i.e. directly above Equator: at the northern horizon

Preparations

Find a spot outside with a good view of the northern sky and the horizon. This activity can be done as soon as the North Star is visible. Therefore, the summer time may not the best season for this activity.

Finding Polaris

Finding Polaris in the sky is rather simple. As soon as the stars are visible, let the students look at them for a while and ask them if they knew the group of stars that is often called the Big Dipper. Its name is different in different cultures (Ladle, Great Chariot, Plough, Drinking Gourd). It is easy to find in the northern hemisphere as it is always above the horizon. A video explains this in detail.

Find North with the Stars - Polaris & Ursa Major - Celestial Navigation (Duration: 11:04) https://youtu.be/n_gT9nBfhfo Figure 3 also shows how Polaris can be found using the Big Dipper (also available as individual images). It is present in the worksheet. Find the box of the stellar group and the two stars at the front (α,β) . Extend the line between them five times and find a moderately bright star. This is Polaris, the North Star.

Measuring the elevation of Polaris

Now the students use the kamal. The cord must be kept straight during the measurements. The board must be held with the smaller edges up and down and perpendicular to the line of sight. Any tilt would compromise the measurement.

As shown in Figure 8 (provided in the worksheet), the lower edge of the kamal must be aligned with the horizon. Then, the length of the cord is modified until the upper edge touches the star. The alignment with the horizon and the star should be checked again.

The students count the number of knots needed to keep the kamal aligned. Counting starts with the knot closest to the board. They may have to interpolate the position between knots. They write down the number and read the corresponding angle from the list in their worksheet. They will have determined the latitude.

The values of the various individuals and groups may differ.

Q: Why are the results not always identical?

A: Some aspects are not perfect (especially knot positions), and different kamal sizes change the perspective a bit. Further, the kamal may not always be held correctly.

Q: How would this affect real navigation on open seas? A: Small errors of a few degrees can lead to course deviations. One degree in latitude corresponds to 60 nautical miles. Repeated measurements and additional information can mitigate this effect.

Analysis

This can be done as homework and checked during the next lesson in school. Let the students check their results with a local map that provides coordinates or online services like Google Maps or Google Earth.

In Google Earth, you can right-click on your location and then click on "What's here?". A small window appears at the bottom of the screen and lists two numbers. The first is the latitude in degrees with decimals. This number is added to the worksheet.

The students may realise that the result differs from their own measurement. Let them write down the underlying reasons.

During the next lesson, let them discuss their results.



- Detailed instructions for building the kamal are included. It is a very simple process. The result of the latitude measurement can be easily checked using online resources. This is also part of the activity.
- The teacher is responsible for providing a basic background on latitudes and longitudes. However, the success of learning can be judged from the questions and answers provided.

- Finding Polaris is a prerequisite for the success of this activity. If this activity is conducted by a group, the students can support each other. In addition, the teacher can guide the students by
 - presenting a planetarium software for practising.
 - taking them to a planetarium.
 - using a laser pointer during the field experiment.

ADDITIONAL INFORMATION

CONCLUSION

The kamal is a navigational tool that was invented by Arab navigators and has been used for many centuries since. This activity uses the example of the kamal to demonstrate how navigation at sea can be successful with some knowledge about astronomy and the stars combined with simple tools. The students learn some major aspects of the history of navigation by applying the basics of maths and astronomy. They build their own kamal and learn how to use it to determine their latitude on Earth by using Polaris as the resting reference point in the sky. With this activity, they get a feeling for what it took to find one's way on the oceans.

CITATION

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